ТОЭ (практика) Симонов Александр Васильевич

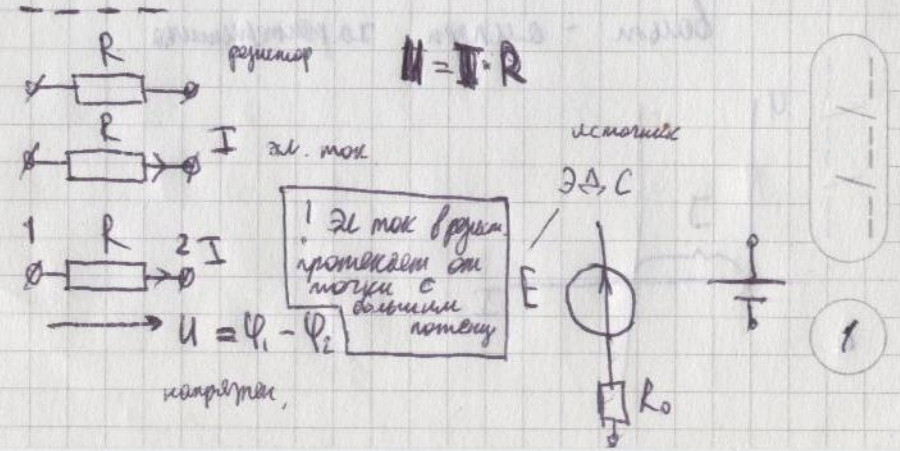
1) учеб. ТОЭ и ЭЦ авт. Бессонов
под ред. Попкин
Атабеков

2) „Основы теории цепей" Страхов, Попкин, Зевеке

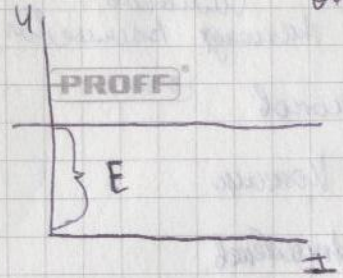
3) методички авт. Атабеков ТОЭ. I часть.
Мелешкин раздел III

4) расчёт задания Атабеков а) „разветв. цепь пост. тока";
б) авт. Кондратьева, Конново, разветвленная цепь синусоидального тока.

5) калькулятор с возможностью операций с матрицами и комплексными числами.



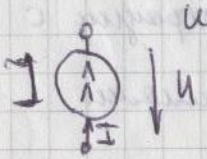
вольт - амперн. характеристика:



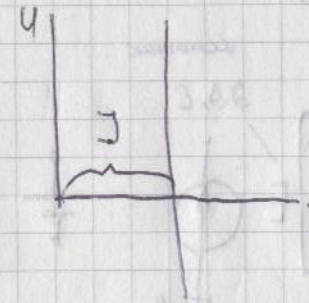
реальная

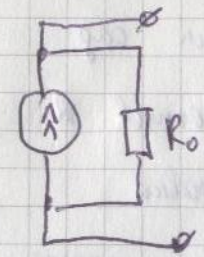
$\varphi_2 = \varphi_1 + E$! стрелка в кружке направлена в сторону повыш. потенц.!

источник тока



вольт - амперн. характеристика





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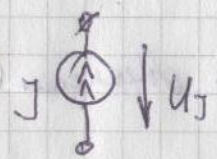
$E = J R_0$

Мощности источников и потреб. в сопротивлениях

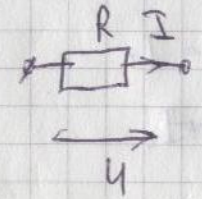
$P_E = E \cdot I$

$I > 0 \Rightarrow P > 0$, источник вырабатывает электроэнергию.

$I < 0 \Rightarrow P <$, источник потребляет электроэнергию.



$P_J = U_J J$



$P = UI = I^2 R = \frac{U^2}{R}$

Энергия - способность тела совершать работу

Мощность - энергия в ед. времени

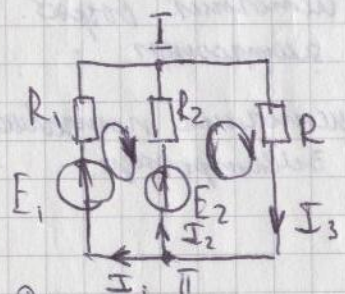
Из з-на сохр. эн. для схем сод. источники ЭДС, тока и сопротив. R следует, что сумма мощностей = сумме мощностей источ. ЭДС и источ. тока.

$$\boxed{\sum P = \sum P_E + \sum P_J}$$ баланс мощностей

2-ны Кирхгофа:

1) $I_1 + I_2 + I_3 = 0$;

2) $\sum E = \sum I \cdot R$



$E_1 = 52$ В; $E_2 = 48$ В;
$R_1 = 4$ Ом; $R_2 = 8$ Ом;
$R_3 = 10$ Ом.

Рассчитать токи, проверить расчёт балансом мощностей

ветви - 3
узлы - 2
N = ветви = 3

(I) $I_1 + I_2 = I_3$

(II) ~~$I_3 = I_1 + I_2$~~

1-й закон Кирхгофа можно сост. для (У - 1) узлов схемы

$N_2 = N - N_1 = 3 - 1 = 2$

$E_1 - E_2 = I_1 \cdot R_1 - I_2 \cdot R_2$

$E_2 = I_2 \cdot R_2 + I_3 \cdot R_3$

$$\begin{cases} I_1 + I_2 - I_3 = 0, \\ 4I_1 - 8I_2 = 4, \\ 8I_2 + 10I_3 = 48, \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 4 & -8 & 0 \\ 0 & 8 & 10 \end{vmatrix} = 1 \cdot (-1)^{1+1} \cdot (-8 \cdot 10) +$$

$$+ 4 \cdot (-1)^{1+2} (1 \cdot 10 - 8 \cdot (-1)) = -80 - 72 = -152$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & -1 \\ 4 & -8 & 0 \\ 48 & 8 & 10 \end{vmatrix} =$$

$= -456$

$\Delta_2 =$

$= -152$

$I_1 = \frac{\Delta_1}{\Delta} = \frac{-456}{-152} = 3\ A$;

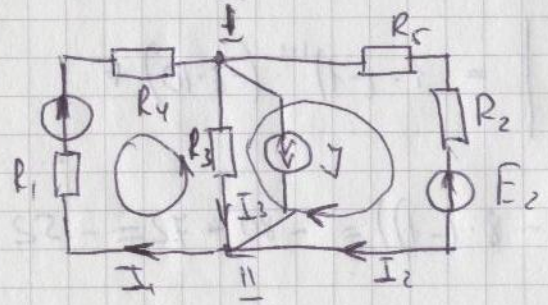
$I_2 = \frac{\Delta_2}{\Delta} = 1\ A$;

$I_3 = I_1 + I_2 = 4\ A$.

$P = I_1^2 \cdot R_1 + I_2^2 \cdot R_2 + I_3^2 \cdot R_3 = 3^2 \cdot 4 + 1^2 \cdot 8 + 4^2 \cdot 10 =$

$= \underline{204\ Вт}$

$P_E = E_1 I_1 + E_2 I_2 = 52 \cdot 3 + 48 \cdot 1 = \underline{204\ Вт}$



$y = 2$ $E_1 = 15\ В$
$E_2 = 30\ В$
$b = 4$ $R_1 = 1\ Ом$
$R_3 = 5\ Ом$
$b_J = 1$ $R_5 = 3\ Ом$

ветвь с источн. тока
$J = 6\ A$, $R_2 = 2\ Ом$;
$R_4 = 4\ Ом$.

$N = b - b_J = 4 - 1 = 3$

I-й у-н. $N_1 = y - 1 = 1$

II-й у-н $N_2 = N - N_1 = 2$

! При сост. ур. по II-му з-ну Кирхгофа рекоменд. не выбирать контуров с источн. тока !

(I) $\Rightarrow$
$$\begin{cases} I_1 = I_2 + I_3 + J \\ E_1 = I_1 R_1 + I_1 R_4 + I_3 R_3 \\ -E_2 = I_2 R_5 + I_2 R_2 - I_3 R_3 \end{cases}$$

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Аналогич. решен. в I-й задаче получаем:

$I_1 = -2A$;

$I_2 = -1A$;

$I_3 = 5A$.

Проверить получ. рез-ты балансом мощностей

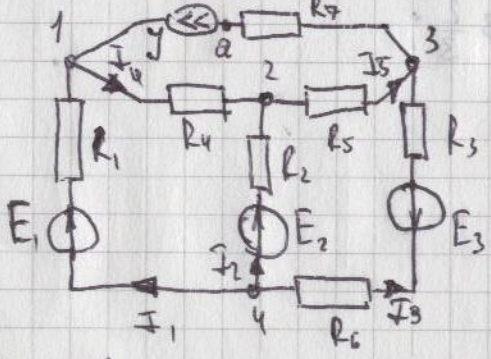
$P = I_1^2 \cdot R_1 + I_1^2 \cdot R_4 \cdot I_3^2 \cdot R_3 + I_2^2 \cdot R_5 + I_2^2 \cdot R_2 =$

$= 4 \cdot 1 + 4 \cdot 4 + 25 \cdot 5 + 1 \cdot 3 + 1 \cdot 2 = \underline{150}$

$P_E = E_1 \cdot I_1 - E_2 \cdot I_2 = 15 \cdot (-2) - 30 \cdot (-1) = 0$

$P_J = U_J \cdot J = I_3 \cdot R_3 \cdot J = 5 \cdot 5 \cdot 6 = 150$

$P = P_E + P_J = 150$



$J = 2A$;
$E_1 = 48\ В$;
$E_2 = 40\ В$;
$E_3 = 10\ В$;

$R_1 = R_6 = 4\ Ом$;
$R_2 = 1\ Ом$
$R_3 = R_7 = R_4 = 2\ Ом$;
$R_5 = 6\ Ом$

$y = 4$ $\varphi_4 = 0$
$b = 6$
$b_J = 1$

$$\begin{cases} \varphi_1 G_{11} + \varphi_2 \cdot G_{12} + \varphi_3 \cdot G_{13} = J_{11} \\ \varphi_1 G_{21} + \varphi_2 \cdot G_{22} + \varphi_3 \cdot G_{23} = J_2 \end{cases}$$

$G_{11} = \frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_7 + \infty} = \frac{3}{4}$

$G_{12} = G_{21} = -\frac{1}{R_4} = -\frac{1}{2}$;

$G_{13} = G_{31} = \frac{1}{R_7 + \infty} = 0$;

$G_{22} = \frac{1}{R_4} + \frac{1}{R_2} + \frac{1}{R_5} = \frac{5}{6}$

$G_{23} = G_{32} = -\frac{1}{R_5} = -\frac{1}{6}$;

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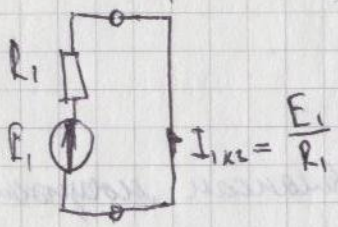
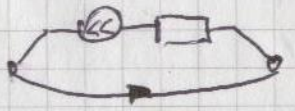
$G_{33} = \frac{1}{R_5} + \frac{1}{R_3+R_6} + 0 = \frac{1}{3}$;

$J_{11} = \frac{E_1}{R_1} + J = 14$;



$I_{1\,кз} = \frac{E_1}{R_1}$



$I_{2\,кз} = J$

$J_{22} = \frac{E_2}{R_2} = 40$;

$J_{33} = -J - \frac{E_3}{R_3+R_6} = -3\frac{2}{3}$.

$$\begin{cases} \frac{3}{4}\varphi_1 - \frac{1}{2}\varphi_2 = 14, \\ -\frac{1}{2}\varphi_1 + \frac{5}{3}\varphi_2 - \frac{1}{6}\varphi_3 = 40, \\ -\frac{1}{6}\varphi_2 + \frac{1}{3}\varphi_3 = -3\frac{2}{3}; \end{cases}$$

$\varphi_1 = 44$ В ; $\varphi_2 = 38$ В ; $\varphi_3 = 8$ В.

$I_1 = \frac{\varphi_4 - \varphi_1 + E_1}{R_7} = 1$ А ; $I_4 = \frac{\varphi_1 - \varphi_2}{R_4} = 3$ А ;

$I_2 = \frac{\varphi_4 - \varphi_2 + E_2}{R_2} = 2$ А ; $I_5 = \frac{\varphi_2 - \varphi_3}{R_5} = 5$ А ;

$I_3 = \frac{\varphi_4 - \varphi_3 - E_3}{R_6 + R_3} = -3$ А ;

Расчёт схемы, провед МУП, проверяют по 1-ому закону Кирхгофа:

для 1 :

$I_1 + J = I_4$;

$1 + 2 = 3$.

Есще один способ проверки – баланс мощностей

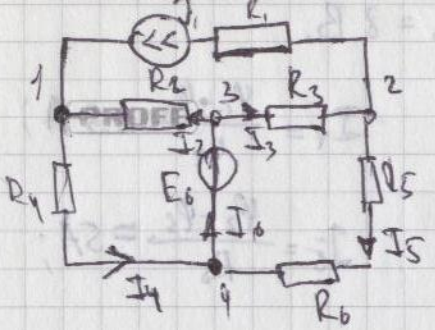
$P_R = I_1^2 \cdot R_1 + I_2^2 R_2 + I_3^2 (R_3 + R_1) + I_4^2 \cdot R_4 + I_5^2 \cdot R_5 + J^2 \cdot R_7 =$

$= 238$ Вт

$P_E = \oplus E_1 \cdot I_1 \oplus E_2 \cdot I_2 \ominus E_3 \cdot I_4 = 98 + 80 - (-30) = 158$ Вт ;

$P_J = (\varphi_1 - \varphi_a) J = (44 - 4) \cdot 2 = 80$ Вт ;

$\varphi_a = \varphi_3 - J R_7 = 8 - 2 \cdot 2 = 4$ А ; $P_E + P_J = P_R$



$b = 6$
$y = 4$

$\varphi_4 = 0$; $\varphi_3 = \varphi_4 + E_6 = 48$,

$$\begin{cases} \varphi_1 \cdot G_{11} + \varphi_2 \cdot G_{12} + 48\, G_{13} = J_{11}, \\ \varphi_1 \cdot G_{21} + \varphi_2 \cdot G_{22} + 48\, G_{23} = J_{22}, \end{cases}$$

$G_{11} = \frac{1}{R_2} + \frac{1}{R_4} + 0 = \frac{1}{12}$; $G_{22} = \frac{1}{R_3} + \frac{1}{R_5 + R_6} + 0 = \frac{1}{3}$

$G_{12} = G_{21} = 0$; $G_{23} = -\frac{1}{R_3} = -\frac{1}{4}$,

$G_{13} = -\frac{1}{R_2} = \frac{1}{3}$;

$J_{11} = J_1 = 4\,A$,

$J_{22} = -J_1 = -4\,A$;

$$\begin{cases} \frac{1}{2}\varphi_1 - \frac{1}{3} \cdot 48 = 4; \\ \frac{1}{3}\varphi_2 - \frac{1}{4} \cdot 48 = -4 \end{cases}$$

$\varphi_1 = 40\,B$; $\varphi_2 = 24\,B$

$I_2 = \frac{\varphi_3 - \varphi_2}{R_2} = 2{,}67\,A$; $I_4 = \frac{\varphi_1 - \varphi_4}{R_4} = 6{,}67\,A$;

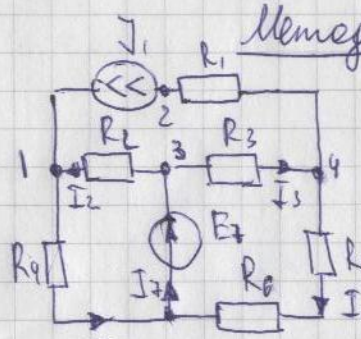
$I_3 = \frac{\varphi_3 - \varphi_2}{R_3} = 6\,A$; $I_5 = \frac{\varphi_2 - \varphi_4}{R_5} = 2\,A$;

$I_6 = \frac{\varphi_4 - \varphi_1 + E_6}{0} = \frac{0}{0}$?

(по закону Кирхгофа) $J_6 = I_4 + I_5 = I_2 + I_3 = 8{,}67\,A$.

Проверка вычислений мощностей

Метод контурных токов:



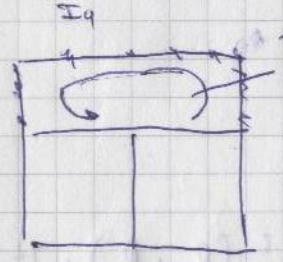
$y = 4$ $b = 6$ $b_J = 1$

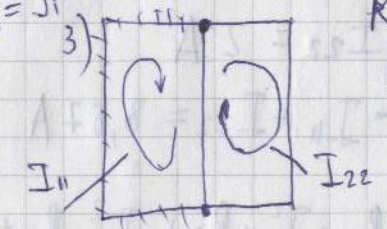
$E_7 = 48\,B$; $J_1 = 4\,A$; $R_1 = 10\,Om$

$R_2 = 3\,Om$; $R_3 = 4\,Om$, $R_4 = 6\,Om$

$R_5 = 8\,Om$

$R_6 = 4\,Om$

1) 

3) 

5) $$\begin{cases} R_{11} I_{11} + R_{12} I_{22} + R_{13} I_{33} = E_{11} \\ R_{21} I_{11} + R_{22} I_{22} + R_{23} I_{33} = E_{22} \end{cases}$$

$R_{11} = R_2 + R_4 = 9\,Om$

$R_{22} = R_3 + R_5 + R_6 = 16\,Om$

$R_{12} = R_{21} = 0\,Om$

$R_{13} = +R_2 = 3\,Om$

$R_{23} = +R_3 = 4\,Om$

$E_{11} = -E_7 = -48\,B$

$E_{22} = +E_7 = 48\,B$

$I_{33} = J_1 = 4$ A

б) $\begin{cases} 9 I_{11} + 0 \cdot I_{22} + 3 \cdot 4 = -48, \\ +16 \cdot I_{22} + 4 \cdot 4 = 48; \end{cases}$

$\begin{cases} I_{11} = -6,67 \text{ A}, \\ I_{22} = 2 \text{ A}; \end{cases}$

$I_2 = -I_{11} - I_{33} = 2,67$ A

$I_5 = I_{22} + I_{33} = 6$ A

$I_4 = -I_{11} = 6,67$ A

$I_{56} = I_{22} = 2$ A

$I_7 = -I_{11} + I_{22} = 8,67$ A

$P_R = I_2^2 R_2 + I_3^2 R_3 + I_4^2 R_4 + I_{56}^2 (R_5 + R_6) + J_1^2 R_1 =$

$= 640,32$ Вт.

$P_{ист} = P_E + P_J$, $P_E = E_7 \cdot I_7 = 416,16$ Вт

[+, т.к. совпадают]

$P_J = J(\varphi_1 - \varphi_2)$

Пусть $\varphi_1 = 0$, $\varphi_3 = \varphi_1 + I_2 \cdot R_2$

$\varphi_4 = \varphi_3 - I_3 \cdot R_3$,

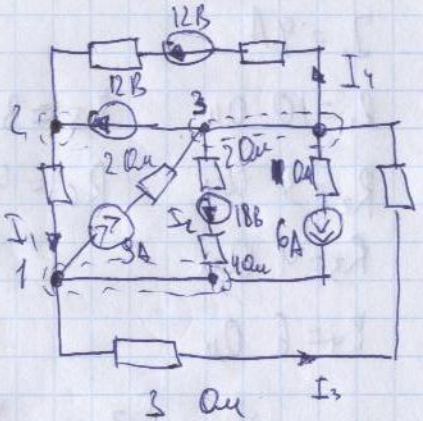
$\varphi_2 = \varphi_4 - J_1 \cdot R_1$

$\varphi_1 - \varphi_2 = 55,99$ В $\quad P_J = 4 \cdot 55,99 = 223,96$ Вт

$640,32 = 416,16 + 223,96$

$= 640,12$

$\Delta\% = \frac{640,32 - 640,12}{640,12} \cdot 100\% = 0,031\%$



$y = 3$

Пусть $\varphi_3 = 0$

$\varphi_2 = \varphi_3 + 12\text{В} = 12\text{В}$

$\varphi_1 \cdot G_{11} + 12 G_{12} = J_{11}$

$G_{11} = \frac{1}{6} + 0 + \frac{1}{(2+4)} + 0 + \frac{1}{6} = \frac{1}{2}$

$G_{12} = -\frac{1}{6}$

$J_{11} = -9 + \frac{18}{4+2} + 6 = 0$

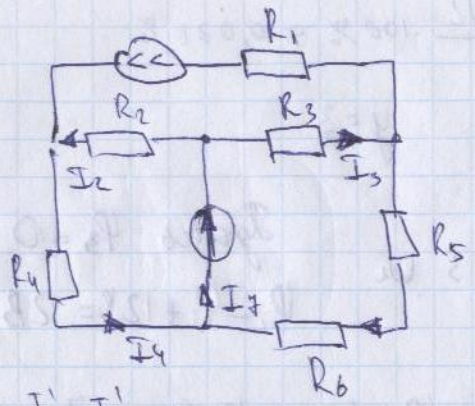
$\frac{1}{2}\varphi_1 - \frac{1}{6} \cdot 12 = 0 \Rightarrow \varphi_1 = 4$ В

$I_1 = \frac{\varphi_2 - \varphi_1}{6} = 1,33$ А

$I_2 = \frac{\varphi_3 - \varphi_1 + 18}{4+2} = 2,33$ А

$I_4 = \frac{\varphi_3 - \varphi_2 + 12}{2+3} = 0$ А

$I_3 = \frac{\varphi_1 - \varphi_3}{3+3} = 0,67$ А



$E_7 = 48$ В

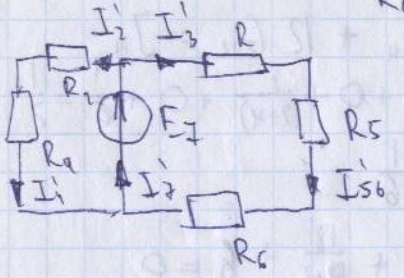
$J_1 = 4$ А

$R_1 = 10$ Ом $\quad R_5 = 8$ Ом

$R_2 = 3$ Ом $\quad R_6 = 4$ Ом

$R_3 = 4$ Ом

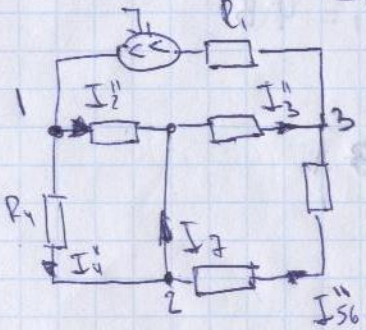
$R_4 = 6$ Ом



$$I_3' = I_{56}' = \frac{E_7}{R_3+R_5+R_6} = \frac{48}{16} = 3 \text{ А}$$

$$I_2' = I_4' = \frac{E_7}{R_2+R_4} = \frac{48}{9} = 5{,}33 \text{ А}$$

$$I_7' = I_4' + I_{56}' = 8{,}33 \text{ А}$$



R_2 и R_4 соед параллельно

$$\begin{cases} U_{12} = I_2'' \cdot R_2 = I_4'' \cdot R_4 \\ I_2'' + I_4'' = J_1 \end{cases}$$

$$I_2'' = J_1 - I_4''$$

$$(J_1 - I_4'')R_2 = I_4'' \cdot R_4$$

$$\boxed{I_4'' = J_1 \frac{R_2}{R_2+R_4}} = 4 \cdot \frac{3}{9} = 1{,}33 \text{ А}$$

$$I_2'' = J_1 \frac{R_4}{R_2+R_4} = 4 \cdot \frac{6}{9} = 2{,}67 \text{ А}$$

(14)

$$I_3'' = J_1 \cdot \frac{R_5+R_6}{R_5+R_6+R_3} = 4\frac{12}{16} = 3 \text{ А}$$

$$I_{56}'' = J_1 \cdot \frac{R_3}{R_3+R_5+R_6} = 1 \text{ А}$$

$$I_7' = I_4'' - I_{56}'' = 1{,}33 - 1 = 0{,}33 \text{ А}$$

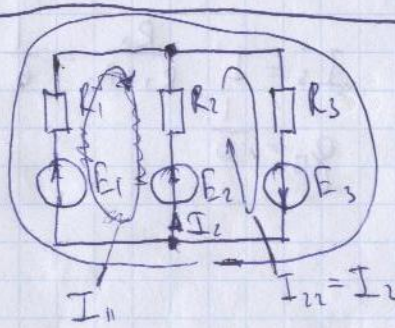
$$I_7 = I_7' + I_7'' = 8{,}66 \text{ А}$$

$$I_4 = I_4' + I_4'' = 6{,}66 \text{ А}$$

$$I_3 = I_3' + I_3'' = 6 \text{ А}$$

$$I_2 = I_2' - I_2'' = 2{,}66 \text{ А}$$

$$I_{56} = I_{56}' - I_{56}'' = 2 \text{ А}$$



$R_1 = R_2 = R_3 = 2$ Ом

g_{11}; g_{12}; g_{13} – ?

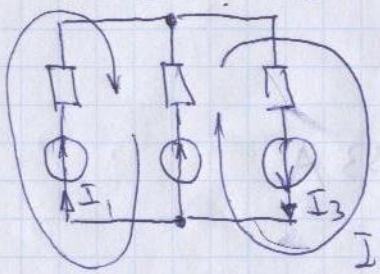
$$\begin{cases} R_{11}I_1 + R_{12}I_{22} = E_{11} \\ R_{21}I_{11} + R_{22}I_{22} = E_{22} \end{cases}$$

$$g_{11} = (-1)^{1+1}\frac{\Delta_{11}}{\Delta} = (-1)^{1+1}\frac{R_{22}}{\begin{vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{vmatrix}} = (-1)^{1+1}\cdot\frac{2+2}{\begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix}} =$$

$$= \frac{4}{12} = \frac{1}{3}.$$

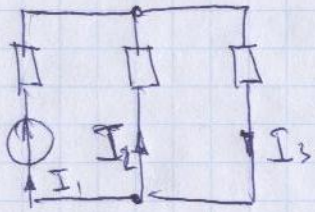
(15)

$g_{12} = (-1)^{1+2} \frac{\Delta_{12}}{\Delta} = (-1)\frac{2}{12} = -\frac{1}{6}$



$g_{13} =$

$I_{22} = I_3$



Пусть $E = 1$ В

$g_{11} = I_1' = \frac{E}{R_{123}} = \frac{1}{3}$

$R_{23} = \frac{R_2 \cdot R_3}{R_2 + R_3} = 1$ Ом

$I_3' = I_1' \frac{R_2}{R_2 + R_3} = \frac{1}{6}$

$g_{13} = \frac{1}{6}$

$R_{123} = R_1 + R_{23} = 3$ Ом

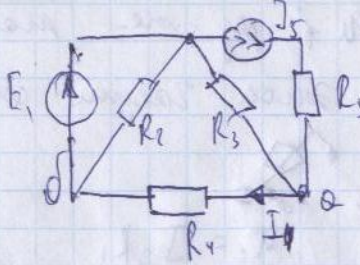
$I_2'' = I_3' - I_1' = -\frac{1}{6} = g_{12}$

$I_1 = g_{11} \cdot E_1 + g_{12} \cdot E_2 + g_{13} \cdot E_3$

$I_2 = g_{21} E_1 + g_{22} E_2 + g_{23} E_3$

$I_3 = g_{31} E_1 + g_{32} E_2 + g_{33} E_3$

Метод эквивалентного генератора

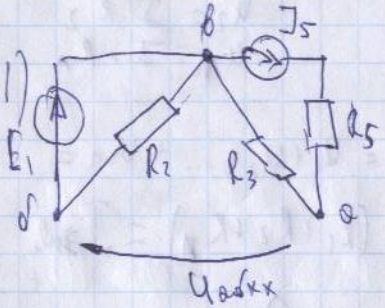


$E_1 = 60$ В

$J_5 = 6$ А $\quad R_2 = R_3 = R_4 = 3$ Ом

$R_5 = 6$ Ом

1)



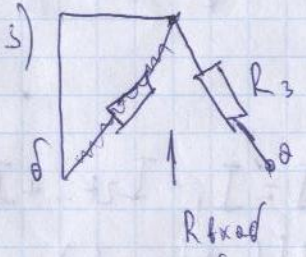
$\varphi_a = const$

$\varphi_в = \varphi_в - J_5 \cdot R_2$

$\varphi_б = \varphi_в - E = \varphi_а - J_5 \cdot R_3 - E$

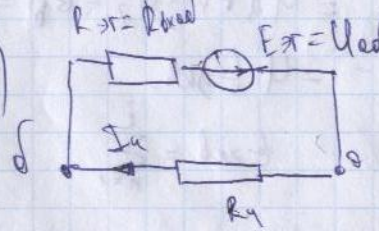
$U_{обхх} = \varphi_а - \varphi_б - J_5 R_3 + E_1 = 6 \cdot 3 + 60 = 78$ В

3)



$R_{вхаб} = R_3$

4)

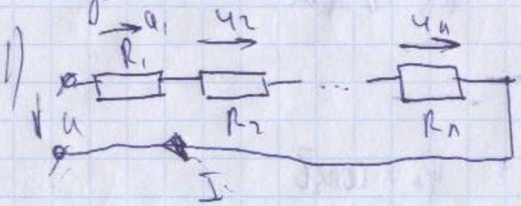


$I_4 = \frac{E_{эг}}{R_{эг} + R_4} = \frac{78}{3+3} = \frac{78}{6} = 13$ А

Эквивалентное преобразование эл. схем

Замена участка эл. цепи группы

участком, наз. эквив., если в рез-те такой замены токораспред. в этой части сис-мы неизменно.

1)  

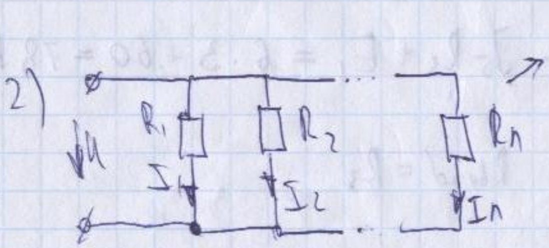
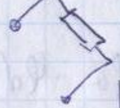
$U_1 = I \cdot R_1$

$U_2 = I \cdot R_2$

$U_n = I \cdot R_n$

$R_{экв} = \sum_{i=1}^{n} R_i$

$U = U_1 + U_2 + \ldots + U_n = I(R_1 + R_2 + R_n) = I R_{экв}$

2)  

$\frac{1}{R} = G$

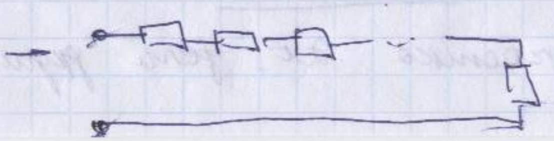
$U_1 = U_2 = \ldots = U_n = U$

$I = I_1 + I_2 + \ldots + I_n = U(G_1 + G_2 + G_n) = U G_{экв}$

$I_1 = \frac{U_1}{R_1} = \frac{U}{R_1} = U G_1$

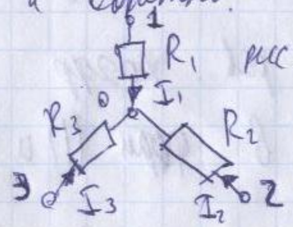
$G_{экв} = \sum_{n=1}^{i}$

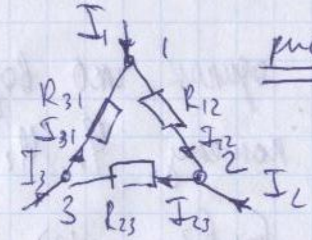
3) 



4) 

5) Преобраз. звезды сопротив. в эквив. треуг. и обратно.

 рис 1

 рис 2

(рис 2) $I_1 = I_{12} - I_{31} = \frac{\varphi_1 - \varphi_2}{R_{12}} - \frac{(\varphi_3 - \varphi_1)}{R_{31}} =$

$= (\varphi_1 - \varphi_2) \cdot G_{12} - (\varphi_3 - \varphi_1) G_{31} = \varphi_1 (G_{12} + G_{31}) - \varphi_2 \cdot G_{12} - \varphi_3 G_{31}$ (1)

(рис 1) $I_1 = (\varphi_1 - \varphi_0) \cdot G_1$; $I_2 = (\varphi_2 - \varphi_0) G_2$; $I_3 = (\varphi_3 - \varphi_0) = G_3$

$I_1 + I_2 + I_3 = 0 = (\varphi_1 - \varphi_0) G_1 + (\varphi_2 - \varphi_0) G_2 + (\varphi_3 - \varphi_0) \cdot G_3$;

$\varphi_0 = \frac{\varphi_1 \cdot G_1 + \varphi_2 \cdot G_2 + \varphi_3 \cdot G_3}{G_1 + G_2 + G_3}$

$I_1 = \left(\varphi_1 - \frac{\varphi_1 G_1 + \varphi_2 \cdot G_2 + \varphi_3 \cdot G_3}{G_1 + G_2 + G_3}\right) G_1 =$

$$= \frac{\varphi_1 g_1 + \varphi_1 g_2 + \varphi_1 g_3 - \varphi_1 g_1 - \varphi_2 g_2 - \varphi_3 g_3}{g_1 + g_2 + g_3} g_1 =$$

$$= \varphi_1 \frac{(g_1 g_2 + g_3 g_1)}{g_1 + g_2 + g_3} - \varphi_2 \frac{g_1 g_2}{g_1 + g_2 + g_3} - \varphi_3 \frac{g_3 g_1}{g_1 + g_2 + g_3} \quad (2)$$

Поки I_1 в схеме звезда и треуг. должна быть одинак, что води. при рав. хозяр при потенц. φ_1, φ_2 и φ_3 в форм (1) и (2)

$$g_{12} = \frac{g_1 g_2}{g_1 + g_2 + g_3} \quad (3)$$

$$g_{31} = \frac{g_3 g_1}{g_1 + g_2 + g_3} \quad (4)$$

$$g_{23} = \frac{g_2 \cdot g_3}{g_1 + g_2 + g_3} \quad (5)$$

} Формулы преобраз. звезды сопротив в эквив Δ

Из уравнений (3), (4), (5), считая g_{12}, g_{31} и g_{23} известн велич., можно получ. формулы для опред g_1, g_2 и g_3

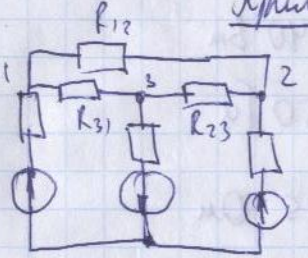
$$R_1 = \frac{1}{g_1} = \frac{R_{12} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}; \quad (6)$$

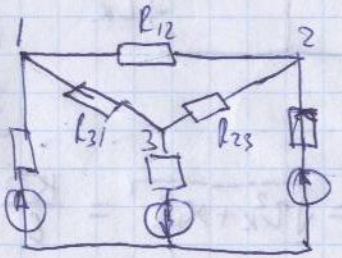
$$R_2 = \frac{1}{g_2} = \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}}; \quad (7)$$

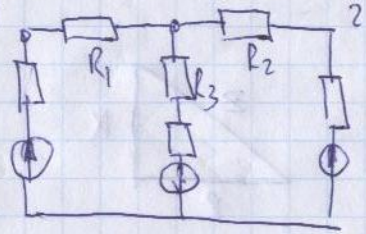
$$R_3 = \frac{1}{g_3} = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} \quad (8)$$

Пример:

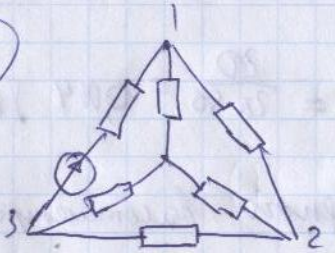
I

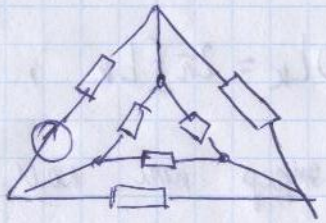


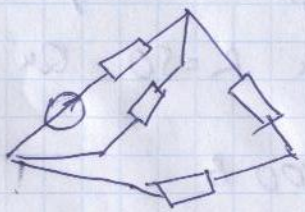


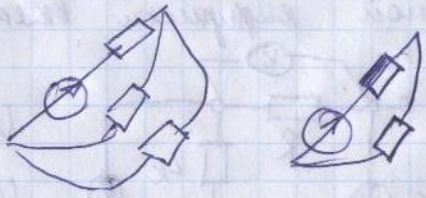


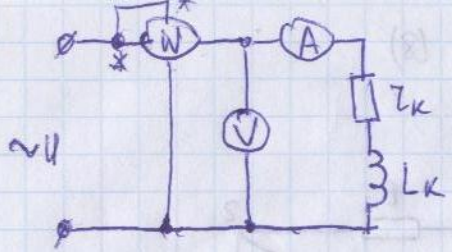
II











$U = 150$ В

$I = 6$ А

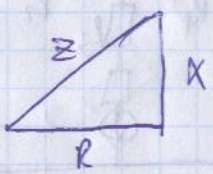
$P = 540$ Вт

$f = 50$ Гц

$\frac{U}{I} = Z = \sqrt{r_к^2 + X_{Lк}^2} = \frac{150}{6} = 25$ Ом

$P = I^2 \cdot r_к$;

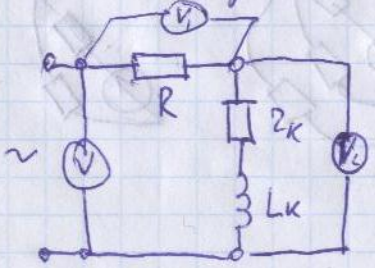
$r_к = \frac{P}{I^2} = \frac{540}{36} = 15$ Ом;



$X_{Lк} = \sqrt{25^2 - 15^2} = 20$ Ом;

$X_{Lк} = \omega L_к = 2\pi f L_к$; $L_к = \frac{20}{2\pi \cdot 50} = 0{,}064$ Гн

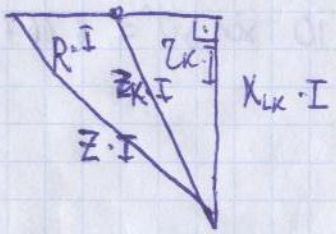
Для опред. при пом. одного вольтметра парам-ов катушки индукт. Комбр. с этой катушкой включили $R = 50$ Ом

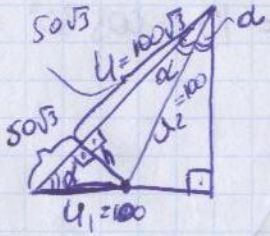


$U_1 = 100$ В

$U_2 = 100$ В

$U = 100\sqrt{3}$ В

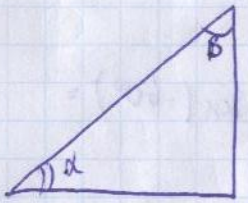


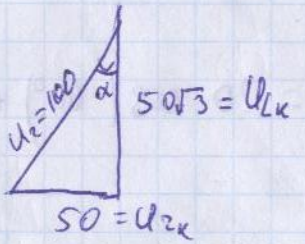


$h = \sqrt{100^2 - (50\sqrt{3})^2} = 50$ Ом

$\alpha = 30°$

$\beta = 60°$





$\frac{R}{r_к} = \frac{U_1}{U_{rк}} \Rightarrow r_к = \frac{U_{rк} \cdot R}{U_1} = \frac{50 \cdot 50}{100} = 25$ Ом

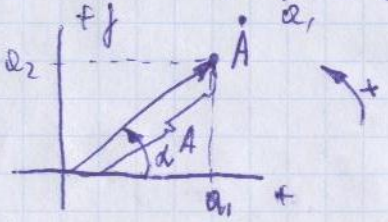
$\frac{U_1}{U_{Lк}} = \frac{IR}{IX_{Lк}} = \frac{R}{X_{Lк}}$; $\Rightarrow X_{Lк} = R\frac{U_{Lк}}{U_1} = 25\sqrt{3}$ Ом.

$X_{Lк} = \omega L_к$; $L_к = \frac{25\sqrt{3}}{2\pi \cdot 50} = 0{,}138$ Гн.

Основные действия с комплексными числами

$j = \sqrt{-1}$; $e^{j\alpha} = \cos\alpha + j\sin\alpha$ - формула Эйлера

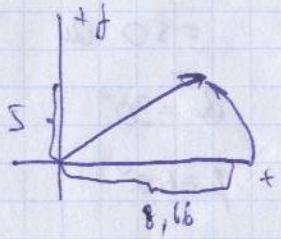
$\dot{A} = Ae^{j\alpha} = \underbrace{A\cos\alpha}_{a_1} + j\underbrace{A\sin\alpha}_{a_2} = a_1 + ja_2$



$A = \sqrt{a_1^2 + a_2^2}$

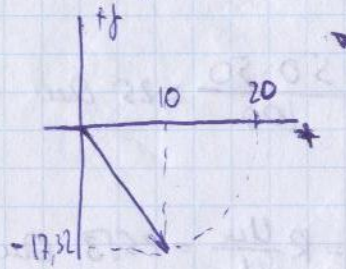
$tg\alpha = \frac{a_2}{a_1}$

1) $10e^{j30°} = 10 \cdot \cos 30° + j\,10 \cdot \sin 30° = 8{,}66 + j5$

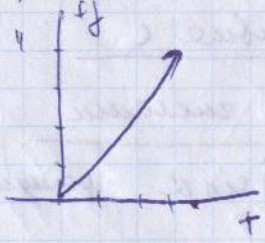


2) $20 \cdot e^{-j60°} = 20 \cdot \cos(-60°) + j\,20 \cdot \sin(-60°) =$

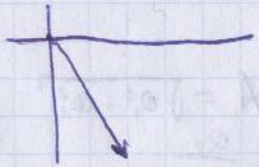
$= 10 - j17{,}32$



3) $3 + j4 = \sqrt{3^2+4^2}\, e^{j \operatorname{arctg} \frac{4}{3}} = 5e^{j53°}$

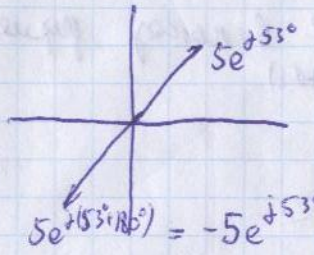


4) $3 - j4 = \sqrt{3^2+(-4)^2}\, e^{j\,-53°}$



5) $5e^{-j53°} = 5e^{j(-53°+360°)} = 5e^{j307°}$

6) $5e^{j53°}$

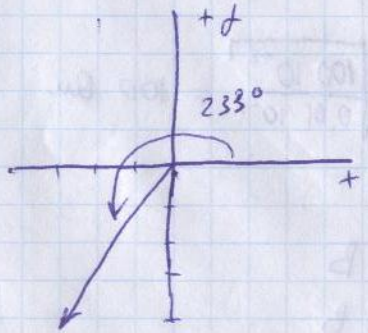


7) ~~$-3 + j4 = \sqrt{(-3)^2+4^2}\, e^{j} = 5e^{j53°}$~~ !!!

$-3 + j4 = -(3 - j4) = -\sqrt{3^2+(-4)^2}\, e^{j \operatorname{arctg} \frac{-4}{3}} =$

$= -5e^{-j53°} = 5e^{j(-53°+180°)} = 5e^{j127°}$

8) $-3 - j4 = -(3 + j4) = -5e^{j53} = 5e^{j233°}$!
(см п. 3)



Если перед действ. частью комп. числа знак „−", но он больше по модулю, число в сопр. перев. в показ. форму, как в п. 3) и 4), после чего от „−" можно избавиться, прибавив к углу 180°

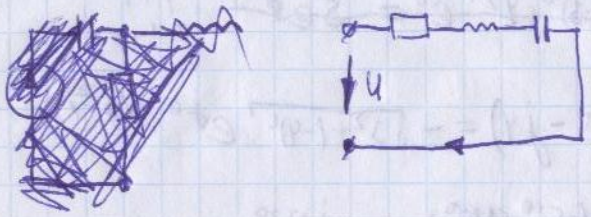
Сложение и вычитание комплексных чисел в алгебраической форме:

$$(a_1 + jb_1) \pm (a_2 + jb_2) = (a_1 \pm a_2) + j(b_1 \pm b_2)$$

Умножение и деление чисел в показательной форме.

$$C_1 e^{j\alpha_1} \cdot C_2 e^{j\alpha_2} = C_1 C_2 e^{j(\alpha_1+\alpha_2)}$$

$$\frac{C_1 e^{j\alpha_1}}{C_2 e^{j\alpha_2}} = \frac{C_1}{C_2} e^{j(\alpha_1-\alpha_2)}$$



$u = 10\sqrt{2}\sin\omega_0 t$

$r = 10$ Ом

$L = 100$ мкГ

$C = 0{,}01$ мкФ

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100\cdot10^{-6}\cdot 0{,}01\cdot10^{-6}}} = 10^6\ c^{-1}$$

$$I_0 = \frac{U}{r} = \frac{10}{10} = 1\ A$$

$$\rho = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}} = \sqrt{\frac{100\cdot10^{-6}}{0{,}01\cdot10^{-6}}} = 100\ Ом$$

$$Q = \frac{\rho}{r} = 10$$

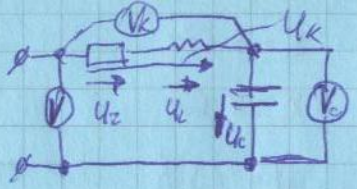
$$U_{L_0} = U\cdot Q = I_0\omega_0 L = 100\ В$$

$$U_{C_0} = U\cdot Q = I_0\cdot\frac{1}{\omega_0 C} = 100\ В$$

$$i = 1\sqrt{2}\sin 10^6 t$$

$$u_L = 100\sqrt{2}\sin(10^6 t + 90°)\ В$$

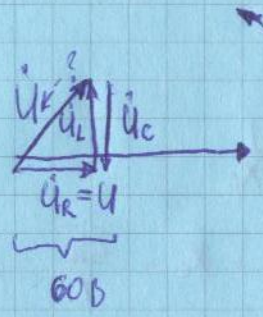
$$u_C = 100\sqrt{2}\sin(10^6 t - 90°)\ В$$



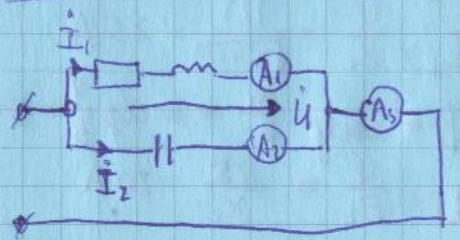
$U = 60$ В

$U_C = 80$ В

Определить U_K, если в цеп. рез. напр.



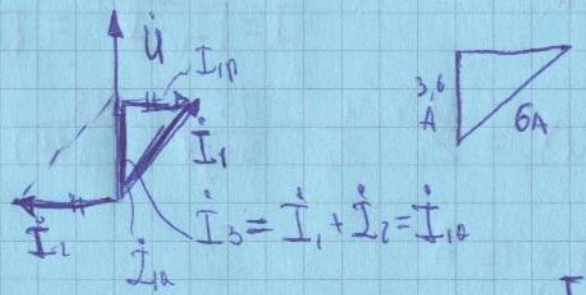
$$U_K = \sqrt{60^2+80^2} = 100\ В$$



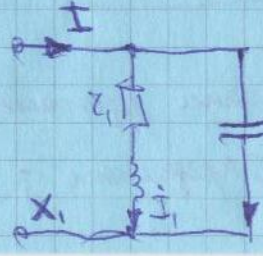
в цепи рез. токов

$I_1 = 6$ А; $I_3 = 3{,}6$ А

$I_2 - ?$



$$I_2 = \sqrt{6^2-3{,}6^2} = 4{,}8$$



1) Подобрать X_C, чтобы цепь оказ. в соот рез. токов.

2) Опред. I_1, I_2, I_3

3) повер. вект. диагр.

4) опред I_{1a}, I_{1p} - ?

$z_1 = 3$ Ом

$X_1 = 4$ Ом

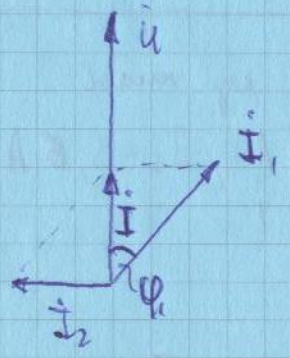
$U = 125$ В

$b_1 + b_2 = 0$; $b_1 = -b_2$

$b_1 = \frac{X_1}{z_1^2 + X_1^2}$

$b_2 = \frac{-X_2}{z_2^2 + (-X_2)^2} = \frac{-X_C}{X_C^2} = -\frac{1}{X_C}$

$\frac{X_1}{z_1^2 + X_1^2} = \frac{1}{X_C}$; $\frac{4}{9+16} = \frac{1}{X_C}$ => $X_C = \frac{25}{4} = 6{,}25$ Ом



$I_1 = \frac{U}{Z_1} = \frac{125}{\sqrt{z_1^2 + X_1^2}} = 25$ А

$\varphi_1 = \operatorname{arctg} \frac{X_1}{R_1} = \operatorname{arctg} \frac{4}{3} = 53°$

$I_2 = \frac{U}{Z_2} = \frac{125}{X_C} = 20$ А

$I = \sqrt{I_1^2 - I_2^2} = \sqrt{25^2 - 20^2} = 15$ А

$\dot{I} = \dot{U}Y_1 + \dot{U}Y_2 = \frac{\dot{U}}{Z_1} + \frac{\dot{U}}{Z_2} = \frac{125}{3+j4} + \frac{125}{(-j6{,}25)} = \ldots = 15$ А

$I_{1a} = I = 15$ А

$I_{1p} = I_2 = 20$ А

$X_L = 3$ Ом

28 $X_C = 8\frac{1}{3}$ Ом

В цепи резонанс токов

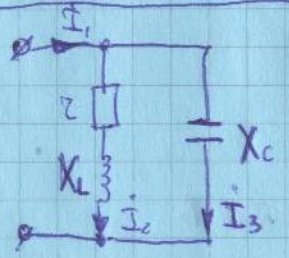
Определить z.

$b_1 + b_2 = 0$

$b_1 = \frac{X_L}{z_1^2 + X_L^2}$; $b_2 = -\frac{1}{X_C}$

$\frac{X_L}{z_1^2 + X_L^2} = \frac{1}{X_C}$ => $z_1^2 + X_L^2 = X_L \cdot X_C$ => $z_1 = 4$ Ом

=> $z_1^2 = X_L \cdot X_C - X_L^2$ => [illegible]



$P = 80$ Вт

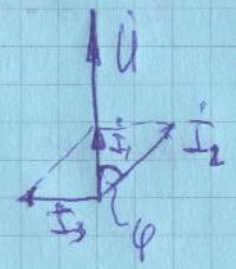
$I_1 = 4$ А

$I_2 = 5$ А

опред: z; X_L; X_C

$P = I_1^2 z$ → $z = \frac{P}{I_1^2} = \frac{80}{25} = 3{,}2$ Ом

$\operatorname{tg} \varphi_2 = \frac{X_L}{z} = \frac{I_3}{I_1} = \frac{3}{4}$

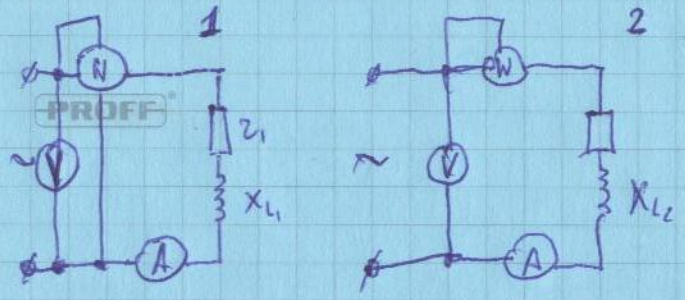


$I_3 = \sqrt{I_2^2 - I_1^2} = \sqrt{25 - 16} = 3$ А

$X_L = \frac{3z}{4} = 2{,}4$ Ом

29

$b_1 = -b_2$ => $\frac{X_L}{z^2 + X^2} = \frac{1}{X_C}$ => $X_C = \frac{3{,}2^2 + 2{,}4^2}{2{,}4} = \frac{9+4}{2{,}4}$



$U_1 = 52$ В

$I_1 = 4$ А

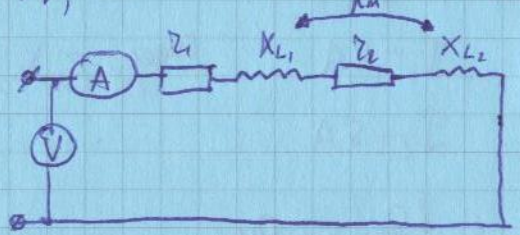
$P_1 = 80$ Вт

$U_2 = 52$ В

$I_2 = 4{,}16$ А

$P_2 = 60{,}5$ Вт

Определ. парам. катушек, соленоид или встречно они подключены катушки в 3-ей схеме. Найти M, если частота 50 Гц



$U = 76$ В

$I = 4$ А

$P_1 = I_1^2 \cdot Z_1 \Rightarrow Z_1 = \frac{P_1}{I_1^2} = \frac{80}{16} = 5$ Ом

$\frac{U_1}{I_1} = Z_1 = \sqrt{Z_1^2 + X_{L_1}^2}$; $Z_1 = 13$ Ом; $X_{L_1} = \sqrt{13^2 - 5^2} = 12$ Ом

По вышел. формулам определяем параметры 2-й катушки.

$Z_2 = 3{,}5$ Ом;

$Z_2 = 12{,}5$ Ом; $X_{L_2} = 12$ Ом

$Z_3 = \frac{U_3}{I_3} = \sqrt{(Z_1+Z_2)^2 + (X_{L_1}+X_{L_2} \pm 2X_m)^2} = \frac{76}{4} = 19$ Ом;

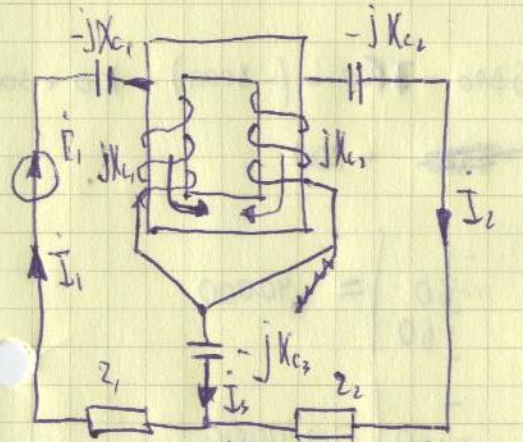
$19 = \sqrt{(5+3{,}5)^2 + (12+12 \pm 2X_m)^2}$;

Анализ последней формулы показывает, что перед X_m возможен минусовый знак $\Rightarrow$ катушки включены встречно.

$(24 - 2X_m) = \sqrt{19^2 - 8{,}5^2} = 17$;

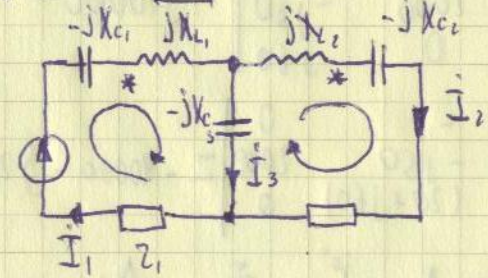
$X_m = 3{,}5$ Ом; $X_m = \omega M = 2\pi f M$;

$M = \frac{3{,}5}{2\pi \cdot 50} \approx 0{,}011$ Гн.



$Z_1 = 5$ Ом
$Z_2 = 20$ Ом
$X_{L_1} = 80$ Ом
$X_{L_2} = 100$ Ом
$X_{C_1} = 20$ Ом
$X_{C_2} = 40$ Ом
$X_{C_3} = 60$ Ом

$X_M = 50$ Ом
$\dot{E} = 2000$ В



$$\begin{cases} \dot{I}_1 - \dot{I}_2 - \dot{I}_3 = 0, \\ \dot{I}_1(Z_1 - jX_{C_1} + jX_{L_1}) - \dot{I}_2 jX_M + \dot{I}_3(-jX_{C_3}) = \dot{E}_3; \\ -\dot{I}_1 jX_M + \dot{I}_2(-jX_{C_2} + Z_2 + jX_{L_2}) - \dot{I}_3(-jX_{C_3}) = 0; \end{cases}$$

$$\begin{cases} \dot I_1 - \dot I_2 - \dot I_3 = 0, \\ \dot I_1 (5+j60) - \dot I_2 j50 + \dot I_3 (-j60) = 1000, \\ -\dot I_1 j50 + \dot I_2 (20+j60) + \dot I_3 j60 = 0; \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & -1 & -1 \\ 5+j60 & -j50 & -j60 \\ -j50 & 20+j60 & j60 \end{vmatrix} = 1 \cdot \begin{vmatrix} -j50 & -j60 \\ 20+j60 & j60 \end{vmatrix} +$$

$$+ 1 \cdot \begin{vmatrix} 5+j60 & -j60 \\ -j50 & j60 \end{vmatrix} + 1 \cdot \begin{vmatrix} 5+j60 & -j50 \\ -j50 & 20+j60 \end{vmatrix} =$$

$$= (3000 - (-j1200 + 3600) + j300 - 3600 - (-3000) + 100 + 300j +$$

$$+ 1200j - 3600 - 2500) = -200;$$

$$\Delta_1 = \begin{vmatrix} 0 & -1 & -1 \\ 1000 & -j50 & -j60 \\ 0 & (20+j60) & j60 \end{vmatrix} = -40000$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & -1 \\ 5+j60 & 1000 & -j60 \\ -j50 & 0 & j60 \end{vmatrix} = j20000$$

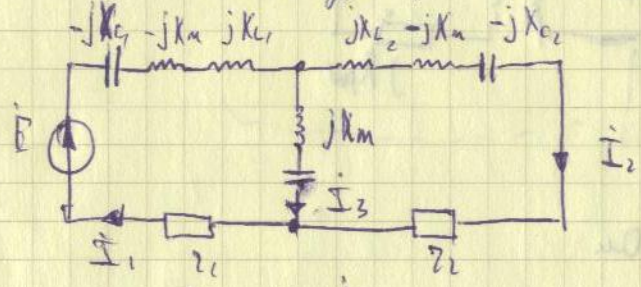
$$\Delta_3 = \begin{vmatrix} 1 & -1 & 0 \\ 5+j60 & -j50 & 1000 \\ -j & (20+j60) & 0 \end{vmatrix} = -40000 - j200000$$

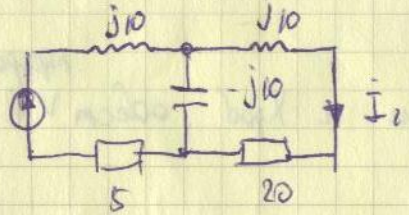
$$\dot I_1 = \frac{\Delta_1}{\Delta} = 200\ A = 200 e^{j0^\circ} \qquad \dot I_3 = \frac{\Delta_3}{\Delta} = 200 + j100\ A =$$

$$\dot I_2 = \frac{\Delta_2}{\Delta} = -j100\ A = 224 e^{j26,5^\circ}$$

$$= 100 \cdot e^{-j90^\circ}$$

Проверим расчёт схемы, используя разбиение магнитных связей





$$\dot\varphi_1 Y_{11} = \dot J_{11}$$

$$Y_{11} = \frac{1}{5+j10} + \frac{1}{-10j} + \frac{1}{20+j}$$

$$= 0,08$$

$$J_{11} = \frac{2000}{5+j10} = \frac{2000 \cdot (5-j10)}{25-100} = 80 - 160j$$

$$\dot\varphi_1 = 1000 - j2000;$$

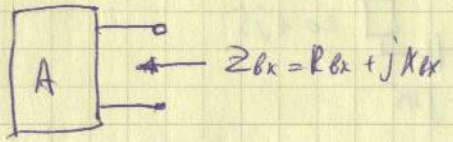
$$\dot I_2 = \frac{\dot\varphi_1 - \dot\varphi_2}{20+j10} = -j100\ A$$

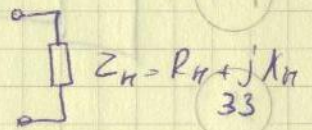
$$\dot I_3 = \frac{\dot\varphi_2 - \dot\varphi_1}{-j10} = 200 + j100\ A$$

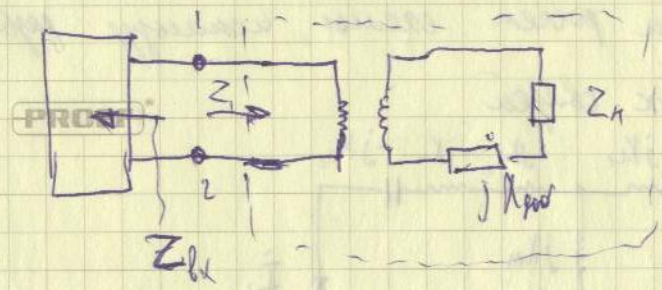
$$\dot I_1 = \frac{\dot\varphi_2 - \dot\varphi_1 + \dot E}{5+j10} = 200 A$$

$$\dot I_2 + \dot I_3 = \dot I_1;$$

$$-j100 + 200 + j100 = 200;$$







$Z_{вх} = 5 + j10$ Ом

$Z_н = 20 + j5$ Ом

Подобр. трансформ. и $X_{доб}$, обесп. V в нагрузке max мощности.

$Z_1 = 5 + j10$

$Z_1 = (20 + j5 + jX_{доб})\left(\frac{W_1}{W_2}\right)^2 = \underbrace{20 \cdot \left(\frac{W_1}{W_2}\right)^2}_{5} + \underbrace{j(5 + X_{доб})\left(\frac{W_1}{W_2}\right)^2}_{-10}$

$20 \cdot \left(\frac{W_1}{W_2}\right)^2 = 5 \; ; \; \left(\frac{W_1}{W_2}\right)^2 = \frac{1}{4}$

$5 + X_{доб} \cdot \frac{1}{4} = -10 \Rightarrow 5 + X_{доб} = -40 \; ;$

$X_{доб} = -45$ - емк. сопр.